OPERATING INSTRUCTIONS

Student Ballistic Pendulum
No. 32140

1. Purpose

You can use the Student Ballistic Pendulum to verify the principle of the conservation of momentum. In so doing you will determine the velocity of a projectile and you will check this determination. This Student Ballistic Pendulum features a design that does not have a rotational component so that the top, bottom, and center of the pendulum have identical degrees of motion in each plane.

In addition, you can remove the firing gun from the apparatus and attach it to an included table clamp that allows you to study other elements of projectile motion. In particular, the dependence of range on initial velocity and on angle of elevation can be examined, and the time of flight may be calculated.
The angle of inclination is adjustable from horizontal to vertical in four increments of 22.5∞.

The initial velocity may be varied by changing the pawl setting on the gun, whether it is set up in the apparatus frame or the table clamp.

2. Description

The apparatus combines a pendulum bob with a spring-loaded plunger or gun for firing a ball-shaped projectile. The pendulum bob is a rubber-lined iron “box” supported by four free-swinging lightweight steel strips. A pin on the bottom of the pendulum bob touches a plastic slide mounted over a millimeter scale. When the ball is shot into the bob, the pin on the bob moves the slide along the scale to indicate the range attained by the projectile. Two extra projectile balls are included.

Besides the pendulum apparatus you will need to have a lab scale that weighs objects up to 1,000 grams, a tape measure, some carbon paper, a spirit level, and a cardboard box. If you have none of these items on hand, we recommend the following:

Description	Catalog No.
Platform Spring Scale	05905
Torpedo Level	88430
Windup Meter Tape	73229-01

3. Setup

There are two different setups: one using the apparatus frame with the included pendulum bob, and one using the table clamp to hold the firing gun.

Apparatus Frame Setup (Fig.1)

Line up the pin on the gun and the two gun mounting screws with the machine-drilled holes on the arm of the apparatus base. Tighten the gun-mounting screws. Attach the pendulum bob to the swinging straps.

Note: The Student Ballistic Pendulum frame must be maintained in a level position in 2 planes for accurate operation. If the unit is not level, the projectile will not be fired in a perfectly horizontal direction, but will be aimed a little above or below the horizontal, complicating calculations for true velocity. Your calculations will also be thrown off if the apparatus is leaning to one side or the other, because the ball will not hit the center of the bob and drive it straight ahead. Instead, it will hit the bob off-center, causing it to sway and twist. Level the apparatus while you are setting it up. Use paper shims under the legs or wooden blocks under the base where needed. You may need to level the apparatus again, so do not strive for perfection.

Table Clamp Setup (Fig.2)

When you are using the table clamp the firing gun should be firmly clamped to a relatively massive table or bench.

Insert the gun centering pin into one of the angle holes and line up one of the gun mounting screws with the center hole of the table clamp and gun mount. Loosely tighten the gun mounting screw. Secure the table clamp to a massive counter, bench or table. Finish tightening the gun mounting screw. Note that this screw may be loosened with the gun still mounted by the centering pin and the angle of inclination changed. You may also orient the gun in the opposite direction by reassembling the gun and the table clamp with the centering pin on the opposite side of the gun.

<u>For Both Setups</u>

In either case, whether you are using the frame or the table clamp setup, a spirit level placed on the firing gun should show it to be horizontal. The gun mounting screws must be tightened to ensure that the gun itself is level.

Fig. 1

Fig. 2

3. Operation

Experiment 1: Pull back the white plastic knob on the spring-coiled lower rod of the gun so that the round latch on the lower rod engages behind the pawl on the upper rod. Thumb-tighten the pawl locking (hex) nut against the pawl. This tightening ensures that every shot contains the same amount of energy. The gun is now triggered. Do not change the trigger position so that you can make several shots with the same force.

Fire a few test shots. If the pendulum tends to sway, it is due to the tilt of the apparatus which places the pendulum off center to the line of flight. To correct the tilt, place paper shims under the legs or wooden blocks under the base where they are needed Keep in mind, however, that because the indicator pin is located very near the center of rotation of the pendulum, a certain amount of sway can be tolerated without noticeably affecting accuracy.

Position the white plastic slide in front of the pin that projects beneath the pendulum bob. (Make sure you don't position the bob within the slide!) Use an erasable marker to mark the original position of the slide on the ruler bed.

Pull up on the release knob to shoot the ball from the spring gun into the suspended pendulum bob The bob will acquire motion from the impact and swing out and upward in the direction of the shot. As the pendulum rises, it loses velocity to the same extent that it would acquire velocity by falling.

Again use the marker to mark the new position of the white plastic slide. The range of the projectile is the difference between the two measurements. This range and the weight of the ball are used to determine the muzzle velocity, explained in the calculations section of this instruction.

To check your velocity determination, remove the pendulum bob and its straps from the apparatus and fire the gun with the pawl in the same setting you have just used. Take your range and altitude measurements and proceed with your calculations.

Note: The muzzle velocity may be changed. Unscrew the pawl locking (hex) nut on the upper rod, and change the position of the pawl. Don't forget to tighten the nut against the pawl when you set the trigger, and your muzzle velocity will remain constant until the next time you set the pawl.

The latch and the pawl are made of hardened steel and will last a long time. Always release the gun in the same manner. A fast release delivers slightly more energy to the projectile than a slow one. Use whichever you wish but be consistent.

Experiment 2: To check your velocity calculation, remove the pendulum bob and its straps from the apparatus. Fire the gun with the same pawl setting that you used in experiment 1.

Experiment 3: Use the table clamp setup for this experiment. Try several shots with the same pawl setting to see how the time of flight and range vary with the angle of inclination.

4. Measurement

IImportant: The pendulum grips the projectile between two strips of rubber. The weight of the pendulum is its own weight plus the projectile plus a small portion of the support straps. The effective length of the pendulum is the distance between the centers of the support pivots. Do not use any other measurement. This is due to the fact that this pendulum, unlike a conventional pendulum, has no rotational component. Top, bottom, and center have identical degrees of motion in each plane.

5. Calculations

By equating the momentum of the ball immediately before impact to the momentum of the system an instant after impact, an equation is set up from which the initial velocity of the ball may be expressed in terms of the easily measurable masses of the ball and pendulum bob and the determination of the height which the bob rises after impact.

This velocity is then checked by removing the pendulum bob and firing the ball horizontally with the pawl set identically to the first firing. The ball's range and vertical distance of fall are both measured and these measurements are used along with the free-fall equation to calculate the velocity. Thus, two different methods of velocity calculation are used.

Calculations for experiment 1:
The momentum **p** of a body is defined as the product of its mass **m** and its velocity **v:**

$$p = mv, \quad (1)$$

where, in different systems of measurement:

	<u>cgs</u>	<u>mks</u>	<u>fps</u>
m	grams	kilograms	pounds
v	cm/sec	meters/sec	feet/sec
p	gram cm/sec	Kg m/sec	lb ft/sec

The principle of the conservation of momentum may be derived from Newton's laws of motion as follows: Imagine an object of mass $\mathbf{m_1}$ moving with a velocity $\mathbf{v_1}$ when it strikes a stationary object of mass $\mathbf{m_2}$. The deceleration of the first object is, as observed by Newton's second law of motion, due to the force $\mathbf{f_1}$ which the second object exerts upon it. The first object, in accordance with the third law of motion, exerts an equal and opposite force $\mathbf{-f_2}$ upon the second body. Representing the respective accelerations by $\mathbf{a_1}$ and $\mathbf{a_2}$**,**

$$f_1 = -f_2 \text{ and } m_1a_1 = -m_2a_2 \quad (2)$$

These forces *necessarily act for the same time interval* , **Dt.** From the definition of acceleration

$$a_1 = F(Dv_1,Dt) \quad \text{and} \quad a_2 = F(Dv_2,Dt) \quad (3)$$

where $\mathbf{Dv_1}$ is the *change in velocity* of the first body and $\mathbf{Dv_2}$ is the *change in velocity* of the second. Substituting these values of the accelerations in Eq. (2)

$$m_1Dv_1 = -m_2Dv_2 \qquad (4)$$

Since the product of mass by change in velocity represents the change in momentum, it follows from Eq. (4) that the loss in momentum of the first body is just equal to the gain in momentum of the second body. In other words, the total momentum of the system has remained constant during the impact.

In the specific inelastic impact case of the ball being fired into the bob, the momentum of the ball just before impact is equal to the combined momenta of the ball and the bob an instant after impact. In the ballistic pendulum used in this experiment the velocity of the pendulum before impact is zero, and hence its momentum before impact is zero. The momentum of the ball before impact is the product of its mass **m** and its initial velocity $\mathbf{v_i}$ just before impact. Since the projectile becomes imbedded in the pendulum bob after impact, the ball and bob an instant after impact have a common or final velocity $\mathbf{v_f}$ and the combined momenta are

$$m\,v_i + Mv_2 = mv_f + M\,v_f \qquad (5)$$

where:
- m = Mass of the ball projectile
- v_i = Velocity of ball projectile
- M = Mass of the pendulum
- v_f = Velocity of pendulum and captured ball
- v_2 =Velocity of the pendulum before impact

Because v_2 is zero, then

$$m\,v_i = (m + M)v_f \qquad (6)$$

The initial velocity, then, may be represented by

$$v_i = F((m + M),m)\ v_f \qquad (7)$$

As a result of the impact the pendulum is thrust both vertically and horizontally through space. You can calculate the velocity of the pendulum and captured ball together, v_f , after you determine the very small vertical displacement of the pendulum.

The kinetic energy of the system an instant after impact must, by the law of the conservation of energy, equal the increase in potential energy gained by the pendulum when it reaches its highest point. By equating the kinetic energy of the combined masses of the ball and the bob **(m + M)** to the gravitational potential energy gained by the system's rise in altitude

$$F((m + M)(v_f)^2,2) = (m + M)gh \quad (8)$$

from which $v_f = R(2gh)$ (9)

where **g** is the acceleration due to gravity and **h** is the maximum height that the bob attains.

h is not easily measured because of its small value; however, if you measure the horizontal movement **D** of the pendulum having a length **L** you can calculate **h**. (See Fig. 3.)

Fig. 3

$$L^2 = (L - h)^2 + D^2 \quad (10)$$

from which $$2Lh = D^2 + h^2 \quad (11)$$

and, because the pendulum has no rotational component, and "swings" through a very small angle, **h** is very small. Therefore, for practical purposes

$$h \text{ ª } F(D^2, 2L) \quad (12)$$

Where did the energy from the gunshot go? The amount of kinetic energy loss in the collision is great! It can be estimated by

$$F(K_f,K_i) = F(F(1,2)(m + M)v_f^2,F(1,2)mv_i^2) \quad (13)$$

where K_i represents the initial kinetic energy fired from the gun and K_f represents the final kinetic energy of the system.

Substituting $$v_i = B(F(m + M,m))v_f \quad (14)$$

for the initial velocity in Eq. (23) we obtain a useful equation for determining the kinetic energy loss:

$$F(K_f,K_i) = F(m, m + M) \quad (15)$$

In practical application of the Student Ballistic Pendulum over 99% of the original kinetic energy in the gun shot is lost to the internal energy of the collision.

Calculations for Experiment 2 (See Fig. 4)

In the second part of this experiment, the initial velocity v_i can be checked by using the measurements of the range **S** and vertical distance **h** of the fall of the ball when it is shot horizontally and allowed to fall to the floor without striking the pendulum bob. The actual motion of the projectile is the combination of its horizontal constant velocity and its downward uniformly accelerated velocity. During the time interval **t** required for the projectile to reach the floor, it moves horizontally through a distance

$$S = vt \qquad (16)$$

and during the same interval, because of the acceleration due to gravity **g,** it falls a distance

$$h = F(1,2)\ gt^2 \qquad (17)$$

By substituting the t in Eq. 14 as the value for **t** in Eq. 13, and using the measured range and vertical component **h,** you can find the initial velocity from

$$v = S\ R(F(g,2h)) \qquad (18)$$

Fig. 4

Calculations for Experiment 3 (See Fig. 5)

The course of a projectile depends mostly on the velocity of the projectile and the angle at which it is launched. In addition, air friction, air velocity and the rotation of the earth influence the path of the projectile. These latter factors are beyond the scope of this experiment, and their effects will be ignored.

If a projectile is aimed directly at its target, it will arrive below the target due to the downward acceleration of gravity. Therefore, part of the initial velocity at which the projectile is fired must be employed to compensate for the effect of gravity. If a projectile is directed at an angle **a** above the horizontal at a velocity **v**, then the vertical component of the velocity is given by

$$v_v = v \sin a \qquad (19)$$

and the horizontal component of the velocity is given by

$$v_h = v \cos a \qquad (20)$$

The vertical component $\mathbf{v_v}$ will diminish due to the effect of gravity. When the vertical velocity is zero, the projectile attains its maximum height, $\mathbf{h_1}$, which is given by

$$h_1 \;=\; F(v^2 \sin^2 a,2g) \qquad (21)$$

where **g** is the acceleration due to gravity.

Having reached its maximum elevation, the projectile will fall due to the acceleration of gravity and continue until it reaches the point of impact.

The elapsed time, $\mathbf{t_1}$**,** during the rise of the projectile is given by

$$t_1 = F(v \sin a,g) \qquad (22)$$

and the elapsed time, $\mathbf{t_2}$, during the fall of the projectile is obtained from

$$h_1 + h_2 = F(1,2)\, g t_2^2 \qquad (23)$$

explicitly:

$$t_2 = R(F(2(h_1 + h_2),g)) \qquad (24)$$

During the time of flight $t_1 + t_2$, the projectile will travel a horizontal distance of

$$D = v \cos a\, (t_1 + t_2) \qquad (25)$$

where
- v = muzzle, or initial, velocity
- a = angle of inclination
- g = acceleration due to gravity
- h_1 = vertical distance from gun to maximum elevation (rise)
- h_2 = vertical distance from gun to target
- $h_1 + h_2$ = vertical fall from maximum elevation to target
- D = range, horizontal distance from gun to target

Fig. 5

Trajectories/ Definitions:
A trajectory is the path described by a projectile moving under the influence of gravity. The measurable quantities involved are:

Range: The horizontal distance traveled by the projectile

Altitude: The highest point reached by the projectile. Actually there are two altitudes, one with reference to the point of departure of the projectile h_1 and the other with reference to the point of arrival $h_{2.}$

Muzzle Velocity or Initial Velocity: The speed at which the ball is originally shot.

Vertical Velocity: That portion of the muzzle (or initial) velocity in a vertical plane.

Angle of Inclination: The angle at which the projectile begins its flight. Horizontal is an angle of zero, straight up an angle of 90∞.

Adjustments: There are a number of variables which complicate the trajectory. These include wind, air friction, rotation of the earth, and projectile spin.

Time of Flight: The sum of the time of rise and time of fall.

6. Maintenance

The Student Ballistic Pendulum needs no special maintenance. If you should experience any difficulty with the pendulum, the table clamp, or any other component, please contact Central Scientific company, giving details of the problem. To ensure better service, please do not return any apparatus to Central Scientific Company until we have sent you authorization.

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